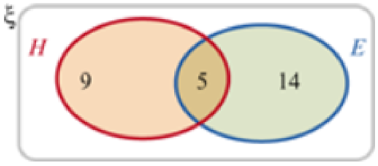
Exercise 2H: Solutions

1 a



Since all students do at least one of these subjects,

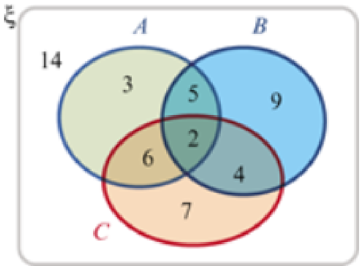
$$\begin{aligned} 9+5+x &= 28 \\ x &= 14 \end{aligned}$$

b i $5+14=19$

ii 9

iii $9+14=23$ or $28-5=23$

2 a

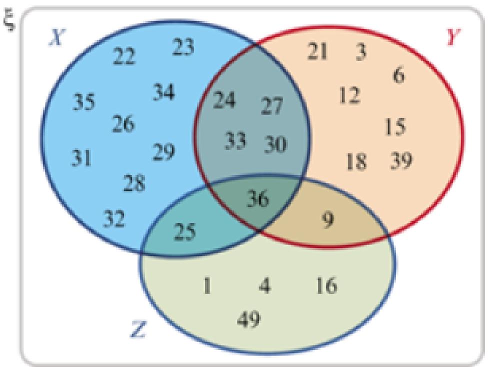


b i $n(A' \cap C') = 9+14 = 23$

ii $$\begin{aligned} n(A \cup B') &= 3+6+5+2+7+14 \\ &= 37 \end{aligned}$$

iii $n(A' \cap B \cap C') = 9$

3



Since 40% don't speak Greek,

$$\begin{aligned} y + 20\% &= 40\% \\ y &= 20\% \end{aligned}$$

Since 40 speak Greek,

$$\begin{aligned} x + 20\% &= 40\% \\ x &= 20\% \end{aligned}$$

20% speak both languages.

4

Since $40-25=15$ don't own a cat,

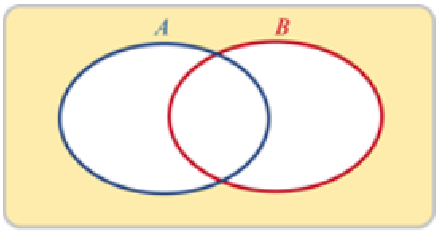
$$\begin{aligned} y + 6 &= 15 \\ y &= 9 \end{aligned}$$

Since 16 own a dog, $x + 9 = 16$

$x = 7$

Seven students own both.

5 a



$(A \cup B)' = A' \cap B'$ is shaded

We must assume every delegate spoke at least one of these languages.

If 70 spoke English, and 25 spoke English and French, 45 spoke English but not French.

$\therefore 45 + 50 = 95$ spoke either English or French or both.

$\therefore 105 - 95 = 10$ spoke only Japanese.

If 50 spoke French, and 15 spoke French and Japanese, 35 spoke French but not Japanese.

$\therefore 35 + 50 = 85$ spoke either French or Japanese or both.

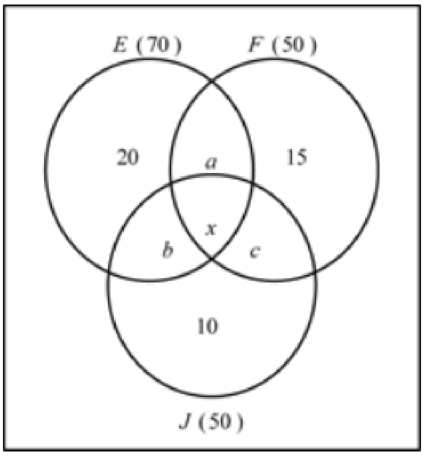
$\therefore 105 - 85 = 20$ spoke only English.

If 50 spoke Japanese, and 30 spoke Japanese and English, 20 spoke Japanese but not English.

$\therefore 20 + 70 = 90$ spoke either Japanese or English or both.

$\therefore 105 - 90 = 15$ spoke only French.

We can now fill in more of the Venn diagram.



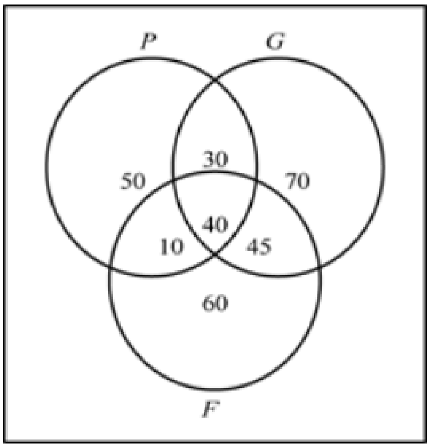
c is the number who don't speak English.

$$\begin{aligned} 105 - 70 &= 10 + c + 15 \\ c + 25 &= 35 \\ c &= 10 \\ x + c &= 15 \\ x &= 5 \end{aligned}$$

5 delegates speak all five languages.

b We have already found that 10 spoke only Japanese.

6 Enter the information into a Venn diagram.

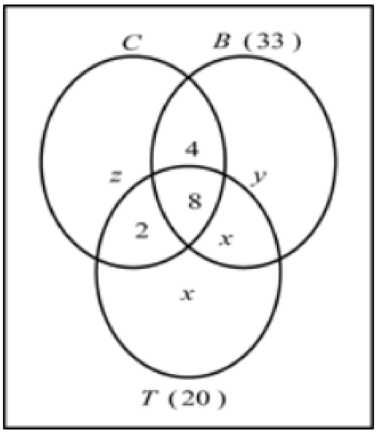


Number having no dessert

$= 350 - 50 - 30 - 70 - 10 - 40 - 45 - 60$

$= 45$

7 Insert the given information on a Venn diagram. Place y as the number taking a bus only, and z as the number taking a car only.



a Using $n(T) = 20,\ 2x + 10 = 20$

$x = 5$

b Using $n(B) = 33$ and $x = 5$,

$12 + 5 + y = 33$

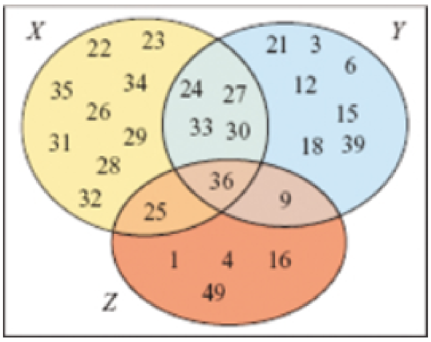
$y = 16$

c Assume they all used at least one of these forms of transport.

$z + 4 + 8 + 16 + 2 + 5 + 5 = 40$

$z = 0$

8 a



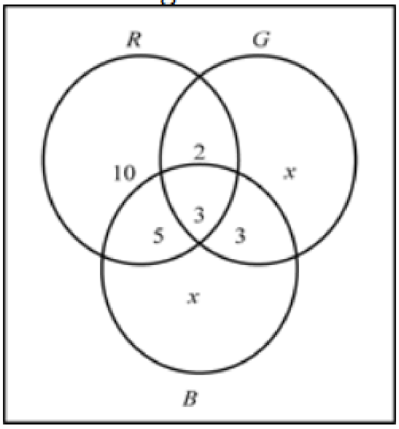
b i $(X \cap Y \cap \mathbb{Z}) =$ intersection of all sets

$= 36$ (from diagram)

ii $|X \cap Y| =$ number of elements in both X and Y

$= 5$ (from diagram)

9 The following information can be placed on a Venn diagram.



The additional information gives $5 > x$ and $x > 3$.

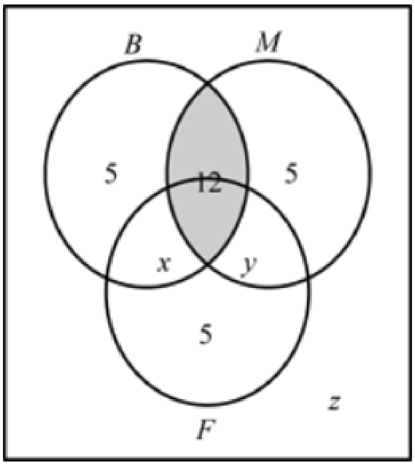
$\therefore \quad x = 4$

Number of students

$$= 10 + 2 + 4 + 5 + 3 + 3 + 4$$
$$= 31$$

20 bought red pens, 12 bought green pens and 15 bought black pens.

10 Enter the given information as below. $B \cap M$ is shaded.

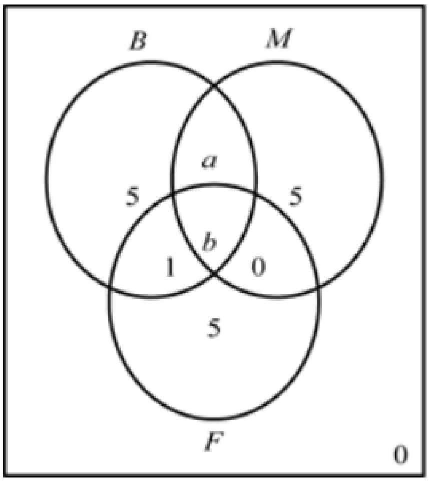


$$5 + 12 + 5 + 5 + x + y + z = 28$$
$$27 + x + y + z = 28$$
$$x + y + z = 1$$

This means that exactly one of x, y and z must equal 1, and the other two will equal zero.

Since $|F \cap B| > |M \cap F|$, the Venn diagram shows that this means $x > y$.

$\therefore \quad x = 1, y = z = 0$



$a + b = 12$

$|M \cap F \cap B| = |F'|$

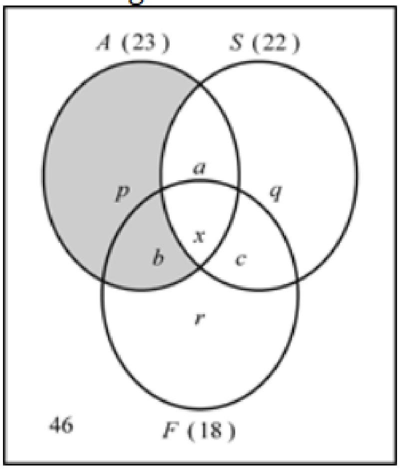
$\therefore \quad b = a + 10$

Substitute in $a + b = 12$:

$$\begin{aligned} a + (a + 10) &= 12 \\ 2a &= 2 \\ a &= 1 \\ b &= a + 10 = 11 \end{aligned}$$

$|M \cap F| = b + 0 = 11$

11 Enter the given information as below.



$a + x = |A \cap S| = 10$

The number of elements in the shaded region is given by

$$\begin{aligned} |A \cap S'| &= |A| - (a + x) \\ &= 23 - 10 \\ &= 13 \\ |A \cup S| &= 10 + 22 \\ &= 32 \end{aligned}$$

$$\begin{aligned} \therefore r + 46 &= 80 - 32 = 48 \\ r &= 2 \end{aligned}$$

Use similar reasoning to show

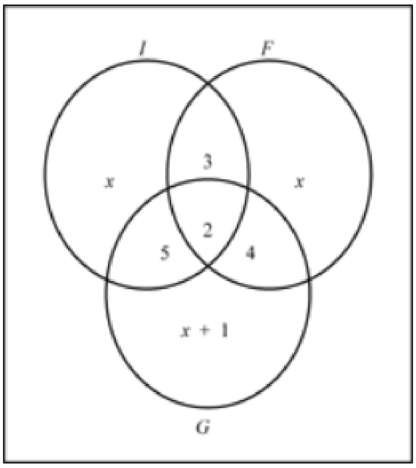
$$\begin{aligned} c + r &= 18 - (b + x) \\ &= 18 - 11 = 7 \end{aligned}$$

Since $r = 2, \ c = 5$

Since $x + c = |S \cup F| = 6$ and $c = 5, \ x = 1$

One person plays all three sports.

12 Enter the information into a Venn diagram.



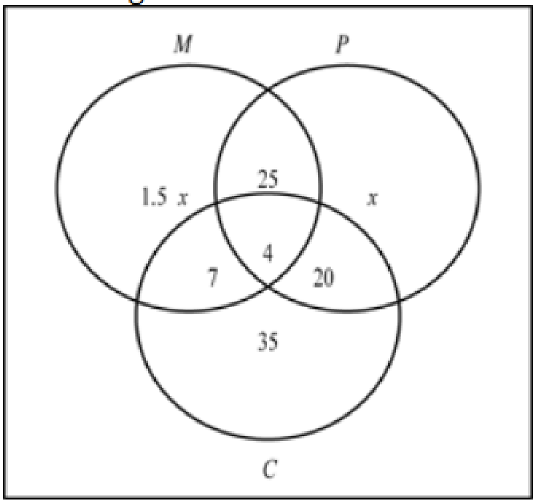
Since they are all proficient in at least one language,

$$\begin{aligned} x+3+x+5+2+4+x+1 &= 33 \\ 3x+15 &= 33 \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

The number proficient in Italian

$$\begin{aligned} &= 6+3+2+5 \\ &= 16 \end{aligned}$$

13 Enter the given information into a Venn diagram.



$$\begin{aligned} 1.5x+25+x+7+4+20+35 &= 201 \\ 2.5x+91 &= 201 \\ 2.5x &= 110 \\ x &= \frac{110}{2.5} \\ &= 44 \end{aligned}$$

The number studying Mathematics

$$\begin{aligned} &= 1.5x+25+7+4 \\ &= 66+25+7+4 \\ &= 102 \end{aligned}$$